

Logarithmic correction to the Brane equation in Topological Reissner-Nordström de Sitter Space

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February 1, 2008

Abstract

In this paper we study braneworld cosmology when the bulk space is a charged black hole in de Sitter space (Topological Reissner-Nordström de Sitter Space) in general dimension, then we compute leading order correction to the Friedmann equation that arise from logarithmic corrections to the entropy in the holographic-braneworld cosmological framework. Finally we consider the holographic entropy bounds in this senario, we show the entropy bounds are also modified by logarithmic term.

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1 Introduction

Holography is believed to be one of the fundamental principles of the true quantum theory of gravity [1, 2]. An explicitly calculable example of holography is the much-studied AdS/CFT correspondence [3]. Unfortunately, it seems that we live in a universe with a positive cosmological constant which will look like de Sitter space-time in the far future. Therefore, we should try to understand quantum gravity or string theory in de Sitter space preferably in a holographic way. Of course, physics in de Sitter space is interesting even without its connection to the real world; de Sitter entropy and temperature have always been mysterious aspects of quantum gravity [4].

While string theory successfully has addressed the problem of entropy for black holes, dS entropy remains a mystery. One reason is that the finite entropy seems to suggest that the Hilbert space of quantum gravity for asymptotically de Sitter space is finite dimensional, [5, 6]. Another, related, reason is that the horizon and entropy in de Sitter space have an obvious observer dependence. For a black hole in flat space (or even in AdS) we can take the point of view of an outside observer who can assign a unique entropy to the black hole. The problem of what an observer venturing inside the black hole experiences, is much more tricky and has not been given a satisfactory answer within string theory. While the idea of black hole complementarity provides useful clues, [7], rigorous calculations are still limited to the perspective of the outside observer. In de Sitter space there is no way to escape the problem of the observer dependent entropy. This contributes to the difficulty of de Sitter space.

More recently, it has been proposed that defined in a manner analogous to the AdS/CFT correspondence, quantum gravity in a de Sitter (dS) space is dual to a certain Euclidean CFT living on a spacelike boundary of the dS space [8] (see also earlier works [9]-[12]). Following the proposal, some investigations on the dS space have been carried out recently [10]-[28]. According to the dS/CFT correspondence, it might be expected that as the case of AdS black holes [29], the thermodynamics of cosmological horizon in asymptotically dS spaces can be identified with that of a certain Euclidean CFT residing on a spacelike boundary of the asymptotically dS spaces.

There has been much recent interest in calculating the quantum corrections to S_{BH} (the Bekenstein-Hawking entropy) [30]-[46]. The leading-order correction is proportional to $\ln S_{BH}$. There are, *two* distinct and separable sources for this logarithmic correction [44, 45] (see also recent paper by Gour and Medved [46]). Firstly, there should be a correction to the number of microstates that is a quantum correction to the microcanonical entropy, secondly, as any black hole will typically exchange heat or matter with its surrounding, there should also be a correction due to thermal fluctuations in the horizon area.

In this paper we consider the brane universe in the bulk background of the topological Reissner-Nordström de Sitter (TRNdS) black holes. In fact there is pressing cosmological motivation for introducing the CFT potential dual to the charge of the black hole. It is, in particular, the presence of a non-vanishing charge that can induce the desirable feature of a non-vanishing bounce [47]. At first we find the thermodynamical quantities of the dual CFT, then we show that the Friedmann brane equation can be in the Cardy-Verlinde formula when the brane crosses the black hole horizon or the cosmological horizon. Taking into account thermal fluctuations defines the logarithmic corrections to both cosmological and black hole horizon entropies. As a result the Cardy-Verlinde formula and Friedmann

brane equation receive the logarithmic corrections.

Therefore here we generalize the logarithmic corrections (with respect to the temperature) that appear in the five dimensional case of Ref [43, 48] to any dimension, but in the specific case of a topological Reissner-Nordstrom black hole in de Sitter (dS) space. The arguments are based on dS/CFT correspondence for the counting of microstates, which of course is not a well established result. However, one may accept this conjecture and study its consequences. Finally we consider the implication of the prior analysis with regard to the holographic entropy bounds, we show the entropy bounds are also modified by logarithmic term.

2 FRW equation in the background of TRNdS Black Holes

The topological Reissner-Nordström dS black hole solution in $(n + 2)$ -dimensions has the following form

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\gamma_{ij}dx^i dx^j, \\ f(r) = k - \frac{\omega_n M}{r^{n-1}} + \frac{n\omega_n^2 Q^2}{8(n-1)r^{2n-2}} - \frac{r^2}{l^2}, \quad (1)$$

where

$$\omega_n = \frac{16\pi G}{n\text{Vol}(\Sigma)}, \quad \phi = -\frac{n}{4(n-1)} \frac{\omega_n Q}{r^{n-1}}, \quad (2)$$

where Q is the electric/magnetic charge of Maxwell field, M is assumed to be a positive constant, l is the curvature radius of de Sitter space, $\gamma_{ij}dx^i dx^j$ denotes the line element of an n -dimensional hypersurface Σ_k with the constant curvature $n(n-1)k$ and its volume $V(\Sigma_k)$. Σ_k is given by spherical ($k = 1$), flat ($k = 0$), hyperbolic ($k = -1$), ϕ is the electrostatic potential related to the charge Q . When $k = 1$, the metric Eq.(1) is just the Reissner-Nordström-de Sitter solution. For general M and Q , the equation $f(r) = 0$ may have four real roots. Three of them are real, the largest one is the cosmological horizon r_c , the smallest is the inner (Cauchy) horizon of black hole, the middle one is the outer horizon r_+ of the black hole. And the fourth is negative and has no physical meaning. The case $M = Q = 0$ reduces to the de Sitter space with a cosmological horizon $r_c = l$. When $k = 0$ or $k = -1$, there is only one positive real root of $f(r)$, and this locates the position of cosmological horizon r_c .

In the case of $k = 0$, $\gamma_{ij}dx^i dx^j$ is an n -dimensional Ricci flat hypersurface, when $M = Q = 0$ the solution Eq.(1) goes to pure de Sitter space

$$ds^2 = \frac{r^2}{l^2}dt^2 - \frac{l^2}{r^2}dr^2 + r^2 dx_n^2, \quad (3)$$

in which r becomes a timelike coordinate.

When $Q = 0$, and $M \rightarrow -M$ the metric Eq.(1) is the TdS (Topological de Sitter) solution [21], which have a cosmological horizon and a naked singularity.

For the purpose of getting the Friedmann-Robertson-Walker (FRW) metric, we impose the following condition [23],

$$\frac{1}{f(r)} \left(\frac{dr}{d\tau} \right)^2 - f(r) \left(\frac{dt}{d\tau} \right)^2 = -1, \quad (4)$$

which leads to a timelike brane. Substituting Eq.(4) into the TRNdS solution Eq.(1), one has the induced brane metric which takes FRW form

$$ds^2 = -d\tau^2 + r^2(\tau)\gamma_{ij}dx^i dx^j, \quad (5)$$

Timelike brane, i.e a brane that has a Minkowskian metric, can only cross the black hole horizon. On the contrary, a spacelike brane, i.e. a brane with Euclidean metric, is able to cross both the black hole horizon and the cosmological horizon. In order to derive the 4-dimensional spacelike brane, the imposed condition (4) has to be slightly changed by replacing the ‘−’ with a ‘+’ on the right-hand side of it.

The equation of motion of the brane is given by[24]

$$\mathcal{K}_{ij} = \frac{\sigma}{n}h_{ij}, \quad (6)$$

where \mathcal{K}_{ij} is the extrinsic curvature, and h_{ij} is the induced metric on the brane, σ is the brane tension. The extrinsic curvature, \mathcal{K}_{ij} , of the brane can be calculated and expressed in term of function $r(\tau)$ and $t(\tau)$. Thus one rewrites the equations of motion (6) as

$$\frac{dt}{d\tau} = \frac{\sigma r}{f(r)}. \quad (7)$$

Using Eqs.(4,7), we can drive FRW equation with $H = \frac{\dot{r}}{r}$,

$$H^2 = \frac{f(r)}{r^2} + \sigma^2 = \frac{-\omega_n M}{r^{n+1}} + \frac{nw_n^2 Q^2}{8(n-1)r^{2n}} + \frac{k}{r^2} - \frac{1}{l^2} + \sigma^2, \quad (8)$$

where, H is the Hubble parameter. We choose the brane tension $\sigma = \frac{1}{l}$ to obtain a critical brane. Therefore Eq.(8) leads to

$$H^2 = \frac{-\omega_n M}{r^{n+1}} + \frac{nw_n^2 Q^2}{8(n-1)r^{2n}} + \frac{k}{r^2}. \quad (9)$$

Making use of the fact that the metric for the boundary CFT can be determined only up to a conformal factor, we rescale the boundary metric for the CFT to be of the following form

$$ds_{CFT}^2 = \lim_{r \rightarrow \infty} \left[\frac{l^2}{r^2} ds_{n+2}^2 \right] = dt^2 + l^2 \gamma_{ij} dx^i dx^j. \quad (10)$$

Evidently, the Euclidean CFT time must be scaled by a factor l/r . Processing on this basis, the thermodynamic relations between the boundary CFT and the bulk TRNdS are given by

$$\begin{aligned} E_{CFT} &= M \frac{l}{r}, & \Phi_{CFT} &= \Phi \frac{l}{r}, \\ T_{CFT} &= T_{TRNdS} \frac{l}{r}, & S_{CFT} &= S_{TRNdS}, \end{aligned} \quad (11)$$

where black hole horizon Hawking temperature T_{TRNdS}^b and entropy S_{TRNdS}^b are given by

$$\begin{aligned} T_{TRNdS}^b &= \frac{f'(r_+)}{4\pi} = \frac{1}{4\pi r_+} \left((n-1) - (n+1) \frac{r_+^2}{l^2} - \frac{nw_n^2 Q^2}{8r_+^{2n-2}} \right), \\ S_{TRNdS}^b &= \frac{r_+^n \text{Vol}(\Sigma)}{4G}, \end{aligned} \quad (12)$$

where $r = r_+$ is black hole horizon and $V_+ = r_+^n \text{Vol}(\Sigma)$ is area of it in $(n+2)$ -dimensional asymptotically dS space.

Here we review the BBM prescription [17] for computing the conserved quantities of asymptotically de Sitter spacetimes briefly. In a theory of gravity, mass is a measure of how much a metric deviates near infinity from its natural vacuum behavior; i.e, mass measures the warping of space. Inspired by the analogous reasoning in AdS space [49, 50] one can construct a divergence-free Euclidean quasilocal stress tensor in de Sitter space by the response of the action to variation of the boundary metric we have

$$T^{\mu\nu} = \frac{2}{\sqrt{h}} \frac{\delta I}{\delta h_{\mu\nu}} = \frac{1}{8\pi G} \left[K^{\mu\nu} - K h^{\mu\nu} + \frac{n}{l} h^{\mu\nu} + \frac{l}{n} G^{\mu\nu} \right], \quad (13)$$

where $h^{\mu\nu}$ is the metric induced on surfaces of fixed time, $K_{\mu\nu}$, K are respectively extrinsic curvature and its trace, $G^{\mu\nu}$ is the Einstein tensor of the boundary geometry. To compute the mass and other conserved quantities, one can write the metric $h^{\mu\nu}$ in the following form

$$h_{\mu\nu} dx^\mu dx^\nu = N_\rho^2 d\rho^2 + \sigma_{ab} (d\phi^a + N_\Sigma^a d\rho) (d\phi^b + N_\Sigma^b d\rho) \quad (14)$$

where the ϕ^a are angular variables parametrizing closed surfaces around the origin. When there is a Killing vector field ξ^μ on the boundary, then the conserved charge associated to ξ^μ can be written as [49, 50]

$$Q = \oint_\Sigma d^n \phi \sqrt{\sigma} n^\mu \xi^\mu T_{\mu\nu} \quad (15)$$

where n^μ is the unit normal vector on the boundary, σ is the determinant of the metric σ_{ab} . Therefore the mass of an asymptotically de Sitter space is as

$$M = \oint_\Sigma d^n \phi \sqrt{\sigma} N_\rho \epsilon \quad ; \quad \epsilon \equiv n^\mu n^\nu T_{\mu\nu}, \quad (16)$$

where Killing vector normalized as $\xi^\mu = N_\rho n^\mu$. Using this prescription [17], the gravitational mass, subtracted the anomalous Casimir energy, of the TRNdS solution is

$$E^c = -M = -\frac{r_c^{n-1}}{\omega_n} \left(k - \frac{r_c^2}{l^2} + \frac{n\omega_n^2 Q^2}{8(n-1)r_c^{2n-2}} \right). \quad (17)$$

The Hawking temperature T_{TRNdS}^c and entropy S_{TRNdS}^c associated with the cosmological horizon are

$$T_{TRNdS}^c = \frac{-f'(r_c)}{4\pi} = \frac{1}{4\pi r_c} \left(-(n-1)k + (n+1)\frac{r_c^2}{l^2} + \frac{n\omega_n^2 Q^2}{8r_c^{2n-2}} \right),$$

$$S_{TRNdS}^c = \frac{r_c^n \text{Vol}(\Sigma)}{4G}, \quad (18)$$

where $V_c = r_c^n \text{Vol}(\Sigma)$ is area of the cosmological horizon. The AD mass of TRNdS solution can be expressed in terms of black hole horizon radius r_+ and charge Q ,

$$E^b = M = \frac{r_+^{n-1}}{\omega_n} \left(1 - \frac{r_+^2}{l^2} + \frac{n\omega_n^2 Q^2}{8(n-1)r_+^{2n-2}} \right). \quad (19)$$

In terms of the energy density $\rho_{CFT} = E_{CFT}/V$, the pressure $p_{CFT} = \rho_{CFT}/n$, the charge density $\rho_{QCFT} = Q/V$ and the electrostatic potential $\Phi_{CFT} = \Phi_r^l$ of the CFT within the volume $V = r^n \text{Vol}(\Sigma)$, also the specific heat of the black hole is given by

$$C^{c,b} = \frac{dE^{c,b}}{dT} = \frac{4\pi r_{c,+}^2 (8k(1-n)l^2 r_{c,+}^{n-2} + 8(n+1)r_{c,+}^n + n\omega_n^2 l^2 r_{c,+}^{-n} Q^2)}{\omega_n (8l^2(n-1)k + 8r_{c,+}^2(n+1) + (1-2n)l^2 \omega_n^2 r_{c,+}^{2-2n} Q^2)}. \quad (20)$$

As one can see the above specific heat is positive in the case $k = -1, k = 0$, for $k = 1$, $C^{c,b}$ is positive only with following condition

$$8(n+1)r_{c,+}^n + n\omega_n^2 l^2 r_{c,+}^{-n} Q^2 > 8k(1-n)l^2 r_{c,+}^{n-2} \quad (21)$$

The first Friedmann equation take the following form

$$H^2 = \frac{16\pi G}{n(1-n)} \left(\rho_{CFT} - \frac{1}{2} \Phi \rho_{QCFT} \right) + \frac{k}{r^2}, \quad (22)$$

3 Logarithmic correction to the Cardy-Verlinde formula and FRW brane cosmology in TRNdS bulk

There has been much recent interest in calculating the quantum corrections to S_{BH} (the Bekenstein-Hawking entropy) [30]-[53]. The corrected formula takes the form

$$\mathcal{S} = S_0 - \frac{1}{2} \ln C + \dots \quad (23)$$

When $r_{c,+}^2 \gg l^2$, $C \simeq nS_0$, in this case we have

$$\mathcal{S} = S_0 - \frac{1}{2} \ln S_0 + \dots \quad (24)$$

It is now possible to drive the corresponding correction to Cardy-Verlinde formula. The Casimir energy E_C , defined as

$$E_C^{c,b} = (n+1)E^{c,b} - nT^{c,b}S^{c,b} - n\phi^{c,b}Q, \quad (25)$$

in this case, is found to be

$$E_C^{c,b} = \frac{-2nkr_{c,+}^{n-1} \text{Vol}(\Sigma)}{16\pi G}, \quad (26)$$

which is valid for both cosmological and black hole horizon. One can see that the entropy Eqs. (18,12) of the cosmological and black hole horizon can be written as

$$S^{c,b} = \frac{2\pi l}{n} \sqrt{\left| \frac{E_C^{c,b}}{k} \right| (2(E^{c,b} - E_q^{c,b}) - E_C^{c,b})}, \quad (27)$$

where

$$E_q^{c,b} = \frac{1}{2} \phi^{c,b} Q = -\frac{n}{8(n-1)} \frac{\omega_n Q^2}{r_{c,+}^{n-1}}. \quad (28)$$

For the present discussion, the total entropy is assumed to be of the form Eq.(24), where the uncorrected entropy, S_0 correspondence to that associated in Eqs. (18,12). It then follows by employing Eqs.(12-19) that the Casimir energy Eq.(25) can be expressed in term of the uncorrected entropy. (Following expressions are valid for both cosmological and black hole horizon, then for simplicity we omit the subscript c and b)

$$E_C = \frac{-2nr_{c,+}^{n-1}\text{Vol}(\Sigma)}{16\pi G} + \frac{nT}{2}LnS_0, \quad (29)$$

After some calculation, the total entropy Eq.(24) to first order in the logarithmic term, is given by [51]

$$S \simeq \frac{2\pi l}{n} \sqrt{\left| \frac{E_C}{k} \right| (2(E - E_q) - E_C)} + \frac{E_q[(3n+1)E - 2nE_q + (1-2n)E_C] + E[nE_C - (n+1)E]}{4E_C(E - E_q - E_C/2)} \\ Ln \left(\frac{2\pi l}{n} \sqrt{\left| \frac{E_C}{k} \right| (2(E - E_q) - E_C)} \right) \quad (30)$$

Therefore taking into account thermal fluctuations defines the logarithmic corrections to both cosmological and black hole entropies. As a result the Cardy-Verlinde formula receive logarithmic corrections in our interest TRNdS black hole background in any dimension. The first Friedmann equation (22) can be rewritten in terms thermodynamical formulas of the CFT on the brane when brane crosses the cosmological or event horizon [24, 26], at these times the first Friedmann equation coincides with the Cardy-Verlind formula. As a direct consequence of the logarithmic correction arising in Eq.(30) the Friedmann equation, also receives the logarithmic correction due to thermal fluctuations of the bulk gravity system. The Hubble parameter H is related with the Hubble entropy as

$$S_H \equiv (n-1) \frac{HV}{4G}, \quad (31)$$

which is equal with bulk black hole entropy at the moment when the brane crosses the black hole horizon $r = r_+$ in the case $k = 1$, and crosses the cosmological horizon $r = r_c$ for the cases $k = 0$ and $k = -1$ [24, 26]. By substituting Eq.(30) into Eq.(31) one finds the modified Friedmann equation at the holographic points

$$H^2 = \frac{16G^2}{(n-1)^2 V^2} S^2 = \frac{16G^2}{(n-1)^2 V^2} \left[\left(\frac{2\pi l}{n} \sqrt{\left| \frac{E_C}{k} \right| (2(E - E_q) - E_C)} \right)^2 + \frac{4\pi l}{n} \sqrt{\left| \frac{E_C}{k} \right| (2(E - E_q) - E_C)} \right. \\ \left. \frac{E_q[(3n+1)E - 2nE_q + (1-2n)E_C] + E[nE_C - (n+1)E]}{4E_C(E - E_q - E_C/2)} \right. \\ \left. Ln \left(\frac{2\pi l}{n} \sqrt{\left| \frac{E_C}{k} \right| (2(E - E_q) - E_C)} \right) \right], \quad (32)$$

after setting $E_q = 0$, $n = 3, k = 1$ the above equation is agree with result of Ref.[48] for Friedmann brane equation in 5-dimensional Schwarzschild de Sitter bulk which is as following

$$H^2 = \left(\frac{2G}{V} \right)^2 \left[\left(\frac{4\pi l}{3\sqrt{2}} \right)^2 \left| E_C \left(E - \frac{1}{2} E_C \right) \right| - \frac{4\pi l}{3\sqrt{2}} \frac{E(4E - 3E_C)}{(2E - E_C) E_C} \right. \\ \left. \sqrt{\left| E_C \left(E - \frac{1}{2} E_C \right) \right|} \ln \left(\frac{4\pi l}{3\sqrt{2}} \sqrt{\left| E_C \left(E - \frac{1}{2} E_C \right) \right|} \right) \right]. \quad (33)$$

At the holographic points $r = r_{c,+}$, after setting $\sigma = \frac{1}{l}$, we have

$$H^2 = \frac{1}{l^2} \quad \text{at} \quad r = r_{+,c}. \quad (34)$$

Formally, the Friedmann equation.(32) holds precisely at the instant when the brane crosses black hole and cosmological horizons. Here we extend the analysis to consider an arbitrary scale factor r where the world-volume of the brane is given by the line-element (5). Thus, around each horizons we assume the Friedmann equation as follows:

$$H^2 = \frac{k}{r^2} + \frac{16\pi G}{n(1-n)} \left(\rho_{CFT} - \frac{1}{2} \Phi \rho_{QCFT} \right) + \frac{32\pi G^2 l}{(n-1)^2 n V^2} \sqrt{\left| \frac{E_C}{k} \right| (2(E - E_q) - E_C)} \\ \frac{E_q[(3n+1)E - 2nE_q + (1-2n)E_C] + E[nE_C - (n+1)E]}{4E_C(E - E_q - E_C/2)} \\ Ln \left(\frac{2\pi l}{n} \sqrt{\left| \frac{E_C}{k} \right| (2(E - E_q) - E_C)} \right), \quad (35)$$

then the logarithmic corrections for the FRW equation are given by the last term on the right-hand side in terms of the uncorrected entropy Eqs.(12,18) of the black hole. Here the logarithmic corrections have been included up to first-order in the logarithmic term. Therefore, at least, the brane receives the thermal radiation from the black hole, the thermal correction should change the dynamics of the brane from the leading order or zero temperature behavior.

Verlinde pointed out that the FRW equation (9) can be related to three cosmological entropy bounds

$$S_{BH} = (n-1) \frac{V}{4GR}, \quad \text{Bekenstein-Hawking bound} \quad (36)$$

$$S_{BV} = -\frac{2\pi R}{n} \left(E - \frac{2\pi G_n Q^2}{(n-1)V} \right), \quad \text{Bekenstein-Verlinde bound} \quad (37)$$

and Hubbel bound which is given by Eq.(31). Here G_n is the gravitational constant in bulk which is given by

$$G_n = \frac{Gl}{n-1}. \quad (38)$$

The FRW equation (9) can be rewritten as

$$S_H = \sqrt{S_{BH}(S_{BH} - 2S_{BV})}, \quad (39)$$

similarly we can rewrite the modified Friedmann Eq.(35) as

$$S_H = \sqrt{S_{BH}(kS_{BH} - 2S_{BV}) + AS_c \ln(S_c)}, \quad (40)$$

where

$$S_c = \frac{2\pi l}{n} \sqrt{\left| \frac{E_c}{k} \right| (2(E - E_q) - E_c)}, \quad (41)$$

$$A = \frac{E_q[(3n+1)E - 2nE_q + (1-2n)E_C] + E[nE_C - (n+1)E]}{4E_C(E - E_q - E_C/2)}. \quad (42)$$

Now if we consider $k = 1$, also if we conjecture the redefined Bekenstein-Hawking entropy as

$$S_{BH} \rightarrow S'_{BH} = S_{BH} - \frac{AS_c \ln S_c}{2(S_{BV} - S_{BH})}, \quad (43)$$

in this case Eq.(40) can be rewritten as following

$$S_H = \sqrt{S'_{BH}(S'_{BH} - 2S_{BV})}, \quad (44)$$

therefore the entropy bounds are also modified by logarithmic term.

4 Conclusion

One of the striking results for the dynamic dS/CFT correspondence is that the Cardy-Verlinde's formula on the CFT-side coincides with the Friedmann equation in cosmology when the brane crosses the cosmological or event horizon $r = r_{c,+}$ of the topological Reissner-Nordström black hole. This means that the Friedmann equation knows the thermodynamics of the CFT (Since conformal symmetry in the bulk is broken by the presence of a black hole, a prospectively dual boundary theory is, strictly speaking, not necessarily a conformal one. Nonetheless, for convenience sake, we continue to refer to the relevant boundary theories as CFT). There is pressing cosmological motivation for introducing the CFT potential dual to the charge of the black hole. Such models are of significant interest because they allow for the possibility of a non-singular bounce (as opposed to a big bang/crunch) [47, 54].

For a large class of black hole, the Bekenstein-Hawking entropy formula receives additive logarithmic corrections due to thermal fluctuations. On the basis of general thermodynamic arguments, Das et al [32] deduced that the black hole entropy can be expressed as

$$\mathcal{S} = \ln \rho = S_0 - \frac{1}{2} \ln (C T^2) + \dots \quad (45)$$

In this paper we have analyzed this correction of the entropy of TRNdS black hole in any dimension in the light of dS/CFT. We have obtain the logarithmic correction to both cosmological and black hole entropies. Then using the form of the logarithmic correction Eq.(24) one can show the corresponding correction to the Cardy-Verlinde formula which relates the entropy of a certain CFT to its total energy and Casimir energy in arbitrary dimension. As a direct consequence of the logarithmic correction arising in Eq.(30) the Friedmann equation, also receives the logarithmic correction due to thermal fluctuations of the bulk gravity system. Moreover, we have consider the holographic entropy bounds in this scenario, we have shown that the entropy bounds are also modified by logarithmic term.

It should be mentioned that in standard cosmology where there are no corrections, the first term in right hand side of equation.(35) represent the curvature contribution to the brane motion. The second term can be regarded as the contribution from the radiation and it redshifts as r^{-4} for a brane moving in the 5-dimensional TRNdS bulk background. The last term in the right-hand side of equation.(35), goes like r^{-6} , it is dominant at early times of the brane evolution while at late times the second term, i.e. the radiative matter term, dominates and thus the last term can be neglected. At this point a couple of questions are raised. First, how the additional term in the Hubble equation.(35) which come from

thermal fluctuations, change the dynamics of the brane? The second question arises when one includes both semiclassical (the self-gravitational effect[55]) and logarithmic corrections. Which is dominant correction and when does this dominant take place during the brane evolution? We hope to address these interesting issues in a future work.

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